MATH 476 – College Geometry

Homework Assignment 1 and Proofs 1 **Solutions**

Homework:

1. **Section 2.1, page 59:** 5, 11, 13

(5) "The medians of a triangle are concurrent lines" translates to, "If three lines are the three medians of a triangle, then they are concurrent." The converse is, "If three (distinct) lines are concurrent, then they are the three medians of a triangle." This is true, although the proof is not easy.

(11) I will just fill in the missing steps: (a) by (1) and Theorem 1; (b) Theorem 2; (c) steps (2) through (5) show that $r \implies s$, so by the definition, a is b.

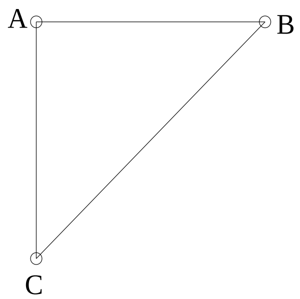
(13) Prove that $p \implies q$.

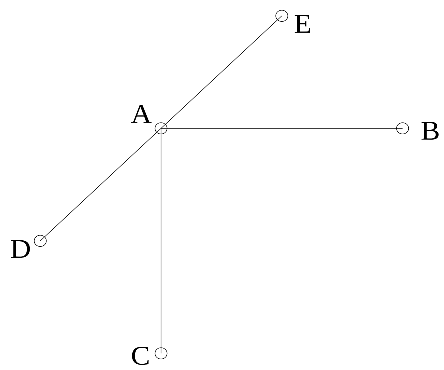
	Conclusions	Justifications
(1)	p	Given
(2)	r or s	These are all of the possibilities
(3)	s	The first possibility
(4)	y	Theorem 3
(5)	$\sim p$	Theorem 1
	Contradiction!	We assumed p
(6)	r	The only remaining possibility
(7)	x	Theorem 2
(8)	q	Theorem 4

Therefore, $p \implies q$.

2. **Section 2.2, page 67:** 6, 9

(6) This is only one pair of possibly many pairs of solutions. Yours may be different.





In the first figure, there are exactly three points, and exactly two points per line. In the second figure, there are 5 points, and one of the lines has three points, while the other two have two points each. Verify for yourself that both pictures satisfy the axioms.

(9) See back of book.

3. **Section 2.3, page 74:** 1, 2, 4, 6

(1) Axiom I-1: We must check that every pair of numbers in S corresponds to a unique line. The pair $\{1,2\}$ belongs to exactly one line: the line $\{1,2,3\}$. The pair $\{1,3\}$ belongs to the same line (and only that line). The pair $\{2,3\}$ also gives that line, and only that line. All of the other pairs appear explicitly in the list of lines (and only appear once).

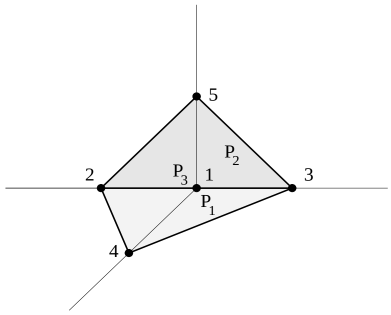
Axiom I-2: The possible sets of three points are: $P_1 = \{1,2,3\}, P_2 = \{1,2,4\}, P_3 = \{1,2,5\}, P_4 = \{1,3,4\}, P_5 = \{1,3,5\}, P_6 = \{1,4,5\}, P_7 = \{2,3,4\}, P_8 = \{2,3,5\}, P_9 = \{2,4,5\}$, and $P_{10} = \{3,4,5\}$. Now it is just a matter of making sure that every set of three (noncollinear) points occurs in exactly one of the 5 specified planes, which I will call $P_1, \ldots P_5$, in the same order they are printed in the book. Then $P_1, P_2, P_4, P_7 \subset P_1; P_1, P_3, P_5, P_8 \subset P_2; P_6 \subset P_3; P_9 \subset P_4; P_{10} \subset P_5$. Thus, every set of three points does give rise to a plane. However, one set of three points seems to give rise to two different planes: The points $\{1, 2, 3\}$ belong to both P_1 and P_2. However, since these points are collinear, the axiom allows them to belong to more than one plane.

Axiom I-3: In our case, this is easy to verify, since all lines except the line $\{1, 2, 3\}$ consist of exactly 2 points, so if two points lie in a plane, then automatically the line determined by them does as well. The only line we must check is the line $\{1, 2, 3\}$. Since the planes P_1 and P_2 both contain all three points, and no other plane contains any two of them, this axiom is also satisfied.

Axiom I-4: We must check all possible intersections of planes. $P_1 \cap P_2 = \{1, 2, 3\}$ is a line. $P_1 \cap P_3 = \{1, 4\}$ is a line. $P_1 \cap P_4 = \{2, 4\}$ is a line. $P_1 \cap P_5 = \{3, 4\}$ is a line. [Bored yet?] $P_2 \cap P_3 = \{1, 5\}$ is a line. $P_2 \cap P_4 = \{2, 5\}$ is a line. $P_2 \cap P_5 = \{3, 5\}$ is a line. $P_3 \cap P_4 = \{4, 5\} = P_3 \cap P_5 = P_4 \cap P_5$ are all lines. Whew!

Axiom I-5: The points $\{1, 2, 3, 4, 5\}$ are noncoplanar (and there are more than 4 of them). Just by looking at the way the lines and planes are defined, we can see that Axiom I-5 is satisfied.

What a hoot! Here's my picture; yours may differ.



(2) I-2, I-3, and I-4 all fail: Points 2, 3, and 4 are non-collinear, but they do not determine a unique plane. For I-3, points 2 and 3 lie in the plane $\{2, 3, 4, 5\}$, but the line $\{1, 2, 3\}$ does not. Finally, the intersection of the first and third planes is $\{2, 3, 4\}$. This contains only part of the line $\{1, 2, 3\}$, and contains the whole line $\{2, 4\}$. This contradicts axiom I-4.

(4) By process similar to what we did in problem 1, we see that axiom I-1 is satisfied (just check each possible pair of points). Each set of three noncollinear points does determine a plane: the possibilities are $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}$, and $\{2, 3, 4\}$. All of them except $\{1, 2, 4\}$ appear in the list of planes, but since 1,2, and 4 are collinear, the points satisfy axiom I-2. However, axiom I-3 fails: the points 1 and 2 lie in the first plane, but the line they determine, $\{1, 2, 4\}$, does not. Axiom I-4 is satisfied since the intersections are all 2-point lines. Axiom I-5 is also satisfied: we have four points, not all coplanar; every line contains at least two distinct points, and every plane contains three noncollinear points.

(6) All axioms are satisfied except I-5 since the 7 points are all coplanar.

4. **Section 2.4, page 87:** 3, 4, 8, 10, 11

(3) (a) See back of book.
(b) $AC + CD = 5 < 6 = AD$.

(4) (a) 2, (b) 5, (c) 1, (d) 3, (e) 4, (f) 3.

(8) (b) $x \leq 3$.
(c) $x \in \mathbb{R}$.

(10) (a) $x \geq -3$.
(b) $x \geq -1$.
(c) $5 \leq x \leq 6$.
(d) $x \geq -3$ or $x \leq 6$. This is equivalent to $x \in \mathbb{R}$.

(11) Since $WZ = 23$ and $XZ = 21$, $WX = 2$. With $WY = 17$, this gives $XY = 15$.